DEPARTMENT OF COMMERCE
SINCLAIR WEEKS, Secretary

WEATHER BUREAU
F. W. REICHELDERFER, Chief

MONTHLY WEATHER REVIEW

Editor, JAMES E. CASKEY, JR.

Volume 81
Number 4

APRIL 1953

Closed June 15, 1953
Issued July 15, 1953

A GENERALIZED STUDY OF PRECIPITATION FORECASTING

PART 1: COMPUTATION OF PRECIPITATION FROM THE FIELDS OF MOISTURE AND WIND

J. C. THOMPSON [1] AND G. O. COLLINS

Short Range Forecast Development Section, U. S. Weather Bureau, Washington, D. C.

[Manuscript received January 26, 1953; Revised April 3, 1953]

ABSTRACT

For the eventual purpose of developing a generalized method for predicting precipitation from assumed accurate prognoses of the required meteorological elements, a preliminary investigation is made of the contemporary relationship between precipitation and the fields of moisture and wind in the atmosphere. Procedures are developed for calculating the rate of precipitation by computing the divergence of the horizontal wind at 50-mb. intervals from the surface to 300 mb. using a technique suggested by Bellamy. From these values, vertical speeds are determined. Using the vertical speeds in Fulks' formula for the rate of precipitation from pseudo-adiabatically ascending air, but with some modifications to compensate for non-saturated initial conditions, a method is derived for calculating the intensity of precipitation. Computed amounts are compared with observed precipitation.

INTRODUCTION

The calculation of precipitation intensity from a knowledge of other atmospheric characteristics is a subject which, in addition to possessing features of purely academic or scientific value, has become of increasing practical interest during the past few years. In part this interest arises because of a gradual increase in the number and accuracy of meteorological observations, which makes feasible the use of more precise methods of calculation; in part it stems from a greater need for quantitative information on precipitation intensities for hydrometeorological work, cloud seeding evaluation, etc.; and in part it stems from the development of new methods for predicting some of the atmospheric variables which are related to precipitation occurrence and intensity as, for example, the use of modern electronic computing equipment to provide a prediction of the height contours at 500 mb. and thus also a prognosis of the "gradient" wind field at that level [1].

The present study was started primarily in an attempt to use these latter developments as a means of forecasting precipitation, under the assumptions (*a*) that the contour fields on constant pressure surfaces will be predicted with increasing efficiency and accuracy as studies such as those making use of electronic computers proceed, (*b*) that other meteorological variables, such as the field of moisture, will be subjected to similar treatment, and (*c*) that it will then be necessary to provide methods for calculating, from these predicted variables, the weather elements about which forecasters and the public in general are primarily concerned—in this case, the occurrence and amount of rainfall. It may be noted in passing that these assumptions are in turn predicated on the hypothesis that it is essential, or at least desirable, that the prediction process should proceed in this somewhat indirect fashion, i. e., that prognostic charts of certain physical quantities, of interest primarily to the meteorologist, should first be provided and that from these data the weather forecast should be made. Whether or not this is the most logical or efficient procedure, or currently provides the most accurate weather prediction, are questions of the basic philosophy of weather forecasting which are beyond the province of this paper to discuss. It is considered sufficient to note here that the construction of such prognostic charts, either mental or formal, is a part of most forecasting procedures at present and probably will continue to be so for some time to come.

The general problem being studied may accordingly be

[1] Present address: Weather Bureau Airport Station, Los Angeles, Calif.

defined: To provide a procedure for computing the amount of precipitation to be expected, having been given a prognosis of those quantities which are related to the precipitation process. As will be apparent from the succeeding discussion, this paper can be considered as only a preliminary report on the progress of work during the first stages of the investigation; plans for continuing the study are discussed briefly at the end of the paper.

METHOD OF ATTACK

In attacking this problem, two types of approach are possible—the statistical and the physical. The statistical approach, in which variables, chosen because synoptic experience and/or physical reasoning indicate that they are related to the element being forecast, are combined graphically or in regression equations, has been used in studies by Carstensen and Hardy [2], Bristor [3], and others.[2] Such procedures are unquestionably useful when the basic relationships between predictor variables and the weather element being predicted are not known quantitatively or when, because of difficulties encountered in solving mathematical expressions, the fundamental equations cannot be used in a practical way. In the approach being used here, however, it is desired to start with whatever basic knowledge is presently available and, by making such assumptions and approximations as are necessary to solve the resulting equations, to build up a generalized relationship between the variables obtained from prognostic charts and the concurrent precipitation. The procedure should therefore be independent of geographic location, season, and similar restrictions which usually apply to statistically derived methods; or if it has initially certain weather or geographical limitations, at least the assumptions which make the restrictions necessary will be known.

In approaching the problem it was considered desirable to limit the study to the computation of precipitation associated with large-scale atmospheric disturbances, i. e., middle latitude cyclones. Such storms are productive of the majority of widespread heavy precipitation situations and are accordingly of considerable importance from both the scientific and economic standpoints. They are also better suited to the type of analysis used here than are small-scale disturbances of the size of single thunderstorms. It was further considered desirable to begin with the fewest assumptions possible, and to assess in turn the validity of each modification which would be required in order to develop an operationally practicable forecasting procedure. Thus, in this study, "true" winds from RAWIN observations are used as a measure of the velocity field, leaving for further investigation an evaluation of the validity of the "gradient" wind and other similar approximations which may be necessary in order to provide a practical forecasting method. It should be noted, however, that some of these latter devices might be expected to filter out some of the small-scale eddy effects which are unavoidably present in RAWIN measurements, and thus may actually provide better approximations of the large-scale precipitation processes than the winds used in this study. Further discussion on this point is given at the end of the paper. Since it has been assumed that the wind and moisture fields will be forecast with sufficient accuracy and sufficiently far in advance to provide reliable predictors for determining the weather to be expected, only the contemporary relationships between these fields and the intensity of precipitation will be considered here.

[2] See adjoining article by Jorgensen.

BASIC ASSUMPTIONS

If it is assumed that cooling processes other than those associated with adiabatic lifting are small, that sufficient and suitable nuclei are present, and that ideal pseudo-adiabatic conditions exist, then the intensity of precipitation may be determined from the fields of moisture and vertical component of velocity. Considering the first of these assumptions, Möller [4] has shown that cooling by radiation from clouds which have an "average distribution" in a low pressure area in middle latitudes is, in the mean, less than 2° C. per day for clouds at altitudes below 25,000 feet. The radiational cooling may be as great as 15° C. per day in the advance portion of a cyclone where the cloud exists as a thin sheet at 25,000–30,000 feet, but thin clouds at this altitude rarely produce precipitation at the surface. For cloud masses with bases below 800 mb. and whose thickness exceeds 200 mb., Hoffer [5] indicates that the rate of cooling is less than 5° C. per day. The rate of cooling due to moderate pseudo-adiabatic lifting, on the other hand, is about 25° C. per day and may exceed 60° C. per day for vertical speeds of 10 cm sec.$^{-1}$ or more. Accordingly, while the effect of radiational cooling is not completely negligible, it may as a first approximation be considered small in comparison with cooling due to ascending motion, especially in those cases where cloudiness is thick enough and exists at sufficiently low levels to produce appreciable precipitation.

Other nonadiabatic temperature changes, i. e., those due to heat exchange between the cloud elements and the air, and heat conduction in the cloud interior, undoubtedly produce structural changes in individual clouds or thunderstorms, but for large-scale processes of the order of a cyclonic disturbance, the effects are unimportant.

The assumption that a sufficient number of nuclei are always present is one which, because of incomplete knowledge of the basic mechanisms of condensation and precipitation in the atmosphere, apparently cannot be adequately evaluated at present. Furthermore, lack of regular observations of the nature and number of atmospheric nuclei which could be used in any systematic way in day-to-day calculation of precipitation, eliminates consideration of this effect in any practical way.

Concerning the final assumption, Möller [6] states that although a "general" pseudo-adiabatic process (i. e., one in which part of the condensed moisture drops out as precipitation and part is carried along in the ascending air) is the normal situation in nature, this process "cannot theoretically be evaluated since no numerical estimate can be made of the amount of precipitation elements which drop out or are carried along." Accordingly, it has been necessary to neglect, in this preliminary study, the problem of moisture storage within the cloud. For a similar reason the evaporation from falling rain has been neglected and the assumption made that all of the water condensed by pseudo-adiabatic cooling is realized in the form of precipitation.

RATE OF PRECIPITATION

The rate of precipitation from a thin layer of saturated air ascending pseudo-adiabatically has been derived by Fulks [7] by assuming that the change in thickness of the rising air may be neglected and by using an approximation for the change in vapor pressure with height in one place in the derivation. The errors which result from either or both of these devices are shown by Fulks to be small in comparison with other factors which affect the pseudo-adiabatic process. The rate of precipitation r is then given by:

$$r=\left[\frac{\epsilon}{RT}\left(\frac{de}{dz}+\frac{eg}{RT}\right)\right]V_z\Delta z=IV_z\Delta z \tag{1}$$

where ϵ is the ratio of the density of water vapor to that of dry air at constant pressure and temperature
R is the gas constant for dry air
T is the mean absolute temperature of the layer
e is the saturation vapor pressure of the layer
z is height
g is the acceleration due to gravity
V_z is the vertical component of velocity of the layer, positive upward
Δz is the thickness of the layer

The quantity I, representing the expression enclosed within the brackets, is the rate of precipitation per unit of vertical speed and thickness of the layer; it is introduced here in order to simplify the notation in the section which follows. Since, for saturated conditions, the vapor pressure is determined by the temperature of the layer and de/dz is determined by its temperature and pressure, the rate of precipitation can be obtained from a knowledge of the temperature, pressure, vertical velocity, and thickness of any shallow layer. Accordingly, if the atmosphere is divided into a number of such layers and the contribution r_i of each layer is computed, the total rate of precipitation P from the column will be,

$$P=\sum_i r_i \tag{2}$$

where index i ranges from the surface to the top of the atmosphere.

Figure 1, adapted from Fulks' paper [7], is a graphical

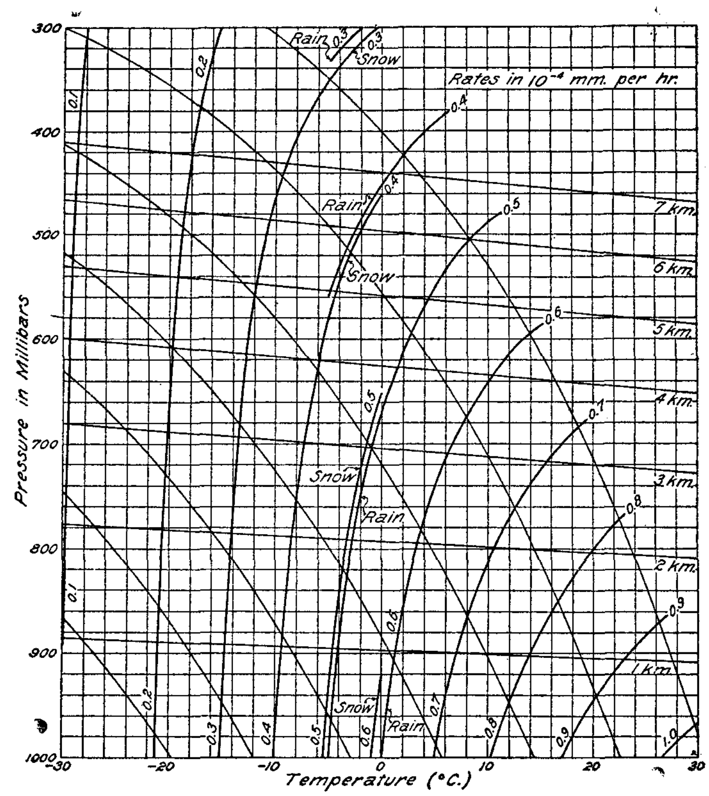

FIGURE 1.—Rates of precipitation from adiabatically ascending air for a one-meter layer with a vertical velocity of one cm. per sec. (After Fulks [7]).

solution of equation (1) for the normal range of atmospheric pressures and temperatures. It provides a method for computing the rate of precipitation from an air column at any instant if the air is assumed to be saturated initially, and the vertical velocity, temperature, pressure, and thickness of each layer in the column are known. Since these latter quantities can be regularly obtained only at finite time intervals in the atmosphere, the convention is adopted in this study that the computed precipitation rates will be used to calculate the amount of precipitation during the 12-hour period between RAOB-RAWIN measurements. Using this device, it is possible to modify Fulks' formula so as to account for, and to some extent eliminate, the assumption of initially saturated conditions.

MODIFICATION FOR NON-SATURATION

The total amount of precipitation, M, contributed by a thin layer during the time, Δt, is

$$M=r\Delta t'=IV_z\Delta z\Delta t' \tag{3}$$

where $\Delta t'$ is the portion of Δt during which the air is saturated. While $\Delta t'$ can be computed from the vertical velocity and moisture available, it is more convenient for computing purposes to change equation (3) to the form

$$M=IV'_z\Delta z\Delta t, \tag{3a}$$

where V'_z is defined by the equation

$$V'_z\Delta t=V_z\Delta t'. \tag{3b}$$

This new "effective speed," V'_z, can be evaluated by assuming that the vertical motion of each layer measured at the time of observation is the mean vertical speed for a time increment between observations. The total ascent, z, of the layer will be

$$z = V_z \Delta t. \tag{4}$$

This total ascent is the sum of the lift needed to reach saturation (z_1) and the lift which produces precipitation (z_2) or,

$$z = z_1 + z_2. \tag{4a}$$

The lift needed to reach saturation is

$$z_1 = -\frac{T_o - T_{do}}{\frac{dT}{dz} - \frac{dT_d}{dz}} \tag{5}$$

where T_d is the dew point temperature of the layer and the subscript "o" denotes the value of the appropriate variable at the initial time. The lift which produces precipitation (z_2) is equal to

$$z_2 = V_z \Delta t'. \tag{6}$$

But by definition (equation 3b), this can be written

$$z_2 = V'_z \Delta t \tag{6a}$$

and the quantity V'_z can be evaluated by combining equations (4) through (6a) giving,

$$V'_z = V_z + \frac{T_o - T_{do}}{\Delta t \left(\frac{dT}{dz} - \frac{dT_d}{dz} \right)}. \tag{7}$$

Substituting in this equation $\Delta t = 12$ hours, $\frac{dT}{dz} = -9.8°$ C. km.$^{-1}$, and $\frac{dT_d}{dz} = -1.6°$ C. km.$^{-1}$ gives for vertical speeds in cm. sec.$^{-1}$,

$$V'_z = V_z - 0.28(T_o - T_{do}). \tag{7a}$$

Introducing these approximations for the adiabatic lapse rate of temperature and the lapse rate of dew point with height introduces only negligible errors in the computations.

VERTICAL VELOCITY

An essential part of the problem is concerned with methods of computing the vertical velocity. Because of its importance in many meteorological processes, a number of investigators have studied procedures for computing vertical motion; and although it would be impractical to discuss the work of all who have studied the problem, it is desired to note that several recent papers have been particularly concerned with attempts to develop relationships between precipitation and vertical motion computed in various ways. The association of vertical motion and precipitation has been studied by Byers and Rodebush [8] who computed the horizontal divergence due to the sea breeze in the Florida Peninsula and showed its relationship to the occurrence of thunderstorms, Miller [9] who, under the assumption of adiabatic motion, related large scale vertical motions to precipitation, Das [10] who has suggested the use of divergence charts for the prediction of precipitation in India, and Bannon [11] who reversed these ideas and obtained an estimate of vertical motion from the rate of rainfall.

In this study, the vertical velocity was obtained by using a method suggested by Bellamy [12]. In this procedure, the horizontal divergence within a traingular area enclosed by three RAWIN (or PIBAL) stations is computed by assuming a linear wind field between points; these results are then used in an equation of continuity, neglecting local changes and horizontal variations in density, to give the vertical velocity at the top of any layer:

$$V_{z2} = \frac{\rho_1}{\rho_2} V_{z1} - \frac{1}{2} \left[\frac{\rho_1}{\rho_2} D_1 + D_2 \right] \Delta z. \tag{8}$$

Here the subscripts "1" and "2" refer to the bottom and top of the layer, respectively, ρ is the density of the air, and D is the horizontal divergence. Neglecting the local change and horizontal variations in density will result in errors of less than two percent in the vertical velocity. Other errors arise because of (*a*) the assumption of linearity in the wind field and (*b*) the inaccuracy and nonrepresentativeness in the observed winds. It should be noted, however, that the assumption of linearity is not completely necessary since the validity of the computation also holds if the mean value of the wind component normal to a side of the triangle is equal to the average of the values at the end points. Variations in the wind of a spatial scale smaller than that of the triangle may be neglected here, since only large scale processes associated with middle-latitude cyclones and anticyclones are being considered.

Errors due to inaccuracies in the wind observations are more important, and are probably of the same order of magnitude as the divergence. However, in the procedure suggested by Bellamy, the vertical velocities are obtained by a process of first summing a group of three partial divergences horizontally and then adding these horizontal divergences through a vertical column. This means that the vertical motion at the top of each layer, and consequently the precipitation contributed by the layer, is determined by at least six wind observations, with the total number of observations used increasing directly as the number of layers used in the computations. Accordingly, if the errors in the observed winds are random, it is possible for the sum of the errors in the computed vertical motions to converge toward zero. In an attempt to check on this possibility, a random difference, i. e., "error" was introduced into the wind direction by using winds smoothed to the nearest of the 16 principal points of the compass. The vertical motions, by layers, computed from these "smoothed" winds showed but little deviation from the vertical velocities obtained using the winds calculated to the nearest degree, thus indicating that these random differences in the winds did indeed converge during the summing process. While it appears reasonable to assume that the actual observational errors in the winds used in the horizontal sums are random, it is probable, on the other hand, that those in the vertical summations are to some extent correlated. Consequently, it is not possible to determine *a priori* whether or not the errors in the computed vertical velocities (and thus the precipitation) will also be small. This must be determined by an examination of the errors in computed precipitation; further discussion of the matter will therefore be deferred until later in the paper.

By noting that $V_z=0$ for a level ground surface, the vertical speed at the top of the first layer, from equation (8), may be written:

$$V_{z1}=-\frac{1}{2}\left[\frac{\rho_o}{\rho_1}D_o+D_1\right](z_1-z_o). \tag{9}$$

Here, and in what follows, the subscript "*o*" refers to the surface, "1" to the top of the first layer, etc. Substituting this expression for V_{z1} in equation (8) and rearranging and collecting terms gives,

$$V_{z2}=-\frac{1}{2}\frac{\rho_o}{\rho_2}D_o(z_1-z_o)-\frac{1}{2}D_2(z_2-z_1)-\frac{1}{2}\frac{\rho_1}{\rho_2}D_1(z_2-z_o). \tag{10}$$

If this process is continued, layer by layer, the general expression for the vertical speed at the top of the i th layer will be seen to be given by,

$$V_{zi}=-\frac{1}{2\rho_i}\left[\rho_o(z_1-z_o)D_o-\rho_i(z_{i+1}-z_i)D_i+\sum_{j=1}^{i}\rho_j(z_{j+1}-z_{j-1})D_j\right]. \tag{11}$$

For the purposes of practical computing, layers 50 mb. in thickness are selected as being sufficiently representative of the vertical distribution of both the wind and moisture fields. It is then assumed that the surface is at 1,000 mb., that the density variation in the vertical is that of the U. S. Standard Atmosphere, and that the divergence at 950 mb. is representative of the layer which extends from the surface to that height. The first of these approximations may produce an extreme error in the computed precipitation of the order of 0.05 inch per 12 hours where the pressure is 25 mb. higher or lower than the assumed value; if the difference exceeds 25 mb., the integration may of course be started at the nearest 50-mb. level. The second assumption introduces a negligible error (less than 1 percent) in the computed precipitation. Use of the last device minimizes undesirable surface turbulence effects which it is desired to eliminate in computing large-scale characteristics of the circulation. The effect of surface friction on the large-scale divergence patterns is assumed small and has been neglected here.

Using the values of ρ and z obtained from the U. S. Standard Atmosphere and performing the summations over 50-mb. intervals, revealed that the terms $\rho_i(z_{i+1}-z_i)$ and $\rho_j(z_{j+1}-z_{j-1})$ in equation (11) were approximately constants, differing only in the third decimal place as i and j varied. Since only two significant figures are justified in the computations, these two terms may be considered as constants and equation (11) then written:

$$V_{zi}=-a_i(D_{950}-D_i)-b_i\sum_{j=950}^{i}D_j,\ j=950, 900, 850, \ldots, i \tag{12}$$

where $a_i=\frac{\rho_i(z_{i+1}-z_i)}{2\rho_i}$ and $b_i=\frac{\rho_j(z_{j+1}-z_{j-1})}{2\rho_i}$. Values of a_i and b_i, when the divergence at each level is obtained in units of sec.$^{-1}$ and the vertical speed in cm. sec.$^{-1}$, are given in table 1.

TABLE 1.—*Values of a and b for computing vertical velocities from equation (12)*

Top of layer (mb.)	*a* (cm.)	*b* (cm.)
300	0.58×10⁵	1.12×10⁵
350	.51	.99
400	.45	.89
450	.41	.81
500	.38	.74
550	.35	.68
600	.33	.64
650	.31	.60
700	.29	.56
750	.27	.53
800	.26	.51
850	.25	.48
900	.24	.47
950	.22	.44

PRACTICAL COMPUTING PROCEDURE

Equations (2), (7a), and (12) provide a method for computing the total amount of precipitation during a 12-hour period, having been given the fields of wind and moisture, and subject to the limitations imposed by the assumptions used in the derivation. Since, as was noted earlier, it is not possible to assess the magnitude of all of the errors introduced by these assumptions, and consequently to evaluate their total effect, it is necessary to apply the procedure to the atmosphere and to examine the data thus obtained. For the purpose of this evaluation, a triangular area in the midwestern United States with vertices at the RAOB-RAWIN stations at Columbia, Mo., Little Rock, Ark., and Nashville, Tenn. was selected. (See fig. 2.) This region is characterized by relatively flat terrain so that the disturbing effects of topography upon the large-scale vertical motions are minimized; it is also located close to the normal tracks of middle-latitude cyclones during the winter season, thus assuring that the observed precipitation would, for the most part, be due to larger scale atmospheric processes; finally, reasonably good data on

FIGURE 2.—Location of the triangular area selected for precipitation computations. The 15 stations in the interior of the triangle were used for verifying the computations.

winds, temperature, and moisture aloft, as well as surface precipitation were available for the area.

The results were verified by using an average of the precipitation recorded at 15 stations within the triangle, with the "mean" value being weighted by selecting more stations near the center than near the edges. (See fig. 2.) This areal distribution was chosen first, because "average" values of divergence and moisture were used in making the computations and the most likely place for the occurrence of these values (and consequently the resulting precipitation) would be near the center of the triangle; and second, because the errors introduced by neglecting the advection of moisture and divergence during the 12-hour period should, on the whole, be smaller near the center of the triangle, if it is assumed that this advection may occur from any direction.

Precipitation rates for an initial sample of one winter month were computed for each 50-mb. layer from the surface to 300 mb. and the divergence was computed from RAWIN data calculated to the nearest degree and in meters per second. Later, however, an examination of the data showed that, in the area selected, the contribution of the layers above 500 mb. during the winter months was usually small and the termination of most RAWIN observations at or below 500 mb. during rain or snow situations frequently necessitated dropping the computations at that level. Furthermore, as was pointed out earlier, the use of wind directions to the nearest 16 compass points (which are easily available from punched-card summaries) introduced no appreciable additional error in the computed precipitation. Consequently, an additional three months of computations were made for the same area using data only to 500 mb. and wind directions smoothed to 16 points of the compass.

An example of a complete computation is shown in tables 2 and 3. In table 2 the divergence, obtained from the winds by using a nomogram adapted from Bellamy [12], is entered in the first column as convergence (convergence=minus divergence) in order to minimize calculations with negative quantities. The last column provides the desired vertical velocities, the entire process representing a solution of equation (12).

If examination of the last column shows negative (downward) vertical speeds at all levels, no further calculations are necessary since the resulting adiabatic processes would tend to inhibit the cooling necessary for precipitation. If, on the other hand, the vertical speed is positive for any layer, it is necessary to see whether or not it is sufficiently large so that, in the 12-hour period between observations, the air will become saturated. This may be done by calculation from equation (7a), the vertical

TABLE 2.—*Example of procedure for computing vertical velocities. Convergence values (C) in the first column are obtained from the divergence (D) of the winds by defining $C=-D$. The last column is the vertical velocity. Data are from the 1500 GMT observations of Dec. 5, 1950*

Pressure (mb.)	C (sec.$^{-1}$)	$C_{950}-C_i$ (sec.$^{-1}$)	$\sum_{j=950}^{i} C_j$ (sec.$^{-1}$)	$a_i(C_{950}-C_i)$ (cm. sec^{-1})	$b_i \sum_{j=950}^{i} C_j$ (cm. sec.$^{-1}$)	V_{zi}: $a_i(C_{950}-C_i)+b_i \sum_{j=950}^{i} C_j$ (cm. sec.$^{-1}$)
300	Missing					
350	11.0×10^{-5}	-11.8×10^{-5}	36.5×10^{-5}	−6.5	36.4	29.9
400	2.4	−4.2	25.5	−1.9	22.7	20.8
450	2.5	−4.3	23.1	−1.8	18.7	16.9
500	3.7	−5.5	20.6	−2.1	15.2	13.1
550	2.5	−4.3	16.9	−1.5	11.5	10.0
600	3.3	−5.1	14.4	−1.7	9.2	7.5
650	2.7	−4.5	11.1	−1.4	6.7	5.3
700	2.8	−4.6	8.4	−1.3	4.7	3.4
750	1.7	−3.5	5.6	−.9	3.0	2.1
800	2.0	−3.8	3.9	−1.0	2.0	1.0
850	3.1	−4.9	1.9	−1.2	.9	−.3
900	.6	−2.4	−1.2	−.6	−.6	−1.2
950	−1.8	0	−1.8	0	−.8	−.8

TABLE 3.—*Example of procedure for computing total precipitation. Values of V_z in the first column are obtained from table 2. Data are from the 1500 GMT observations of Dec. 5, 1950*

Pressure (mb.)	V_z (cm. sec.$^{-1}$)	$0.28(T_o-T_{do})$ (cm. sec.$^{-1}$)	V_z': $V_z-0.28(T_o-T_{do})$ (cm. sec.$^{-1}$)	r' from fig. 1 (mm.hr.$^{-1}$)	Δz (m.)	r: $V_z' r' \Delta z$ (mm. hr.$^{-1}$)
300	Missing					
350	29.9	3.9	26.0	0.01×10^{-4}	990	0.026
400	20.8	3.9	16.9	.01	880	.015
450	16.9	2.8	14.1	.04	805	.045
500	13.1	2.2	10.9	.10	740	.081
550	10.0	1.1	8.9	.18	685	.110
600	7.5	.6	6.9	.29	635	.127
650	5.3	.6	4.7	.35	595	.098
700	3.4	.6	2.8	.40	560	.063
750	2.1	2.0	.1	.37	535	.002
800	1.0	1.1	−.1			
850	−.3					
900	−1.2					
950	−.8					

$\Sigma r = P = 0.567$ (mm. hr.$^{-1}$
$P = .27$ (in. 12-hr.$^{-1}$
Observed Precipitation $P_{obs} = .15$ (in. 12-hr.$^{-1}$

velocities at each level being obtained from table 2, and the values of T_o and T_{do} by averaging the values reported by the RAOB ascents at the vertices of the triangular area selected. If V'_z is negative for all layers, no further steps are necessary since it is presumed that the air will not be lifted sufficiently during the 12-hour period to produce precipitation. However, if V'_z is positive for any layer, it may be substituted for V_z in equation (1) and used to obtain the amount of precipitation contributed by that layer.

In this study, equation (1) was evaluated by making use of the nomogram, figure 1. Precipitation rates were then obtained by making the working assumptions that the initial dew point of the ascending air would be its condensation temperature, and that the initial pressure of the layer would be its condensation pressure. If the air is close to saturation, these assumptions introduce negligible errors, and if the air is far from saturation there will usually be insufficient lift to produce saturation and no precipitation will be computed in any case. However, for high

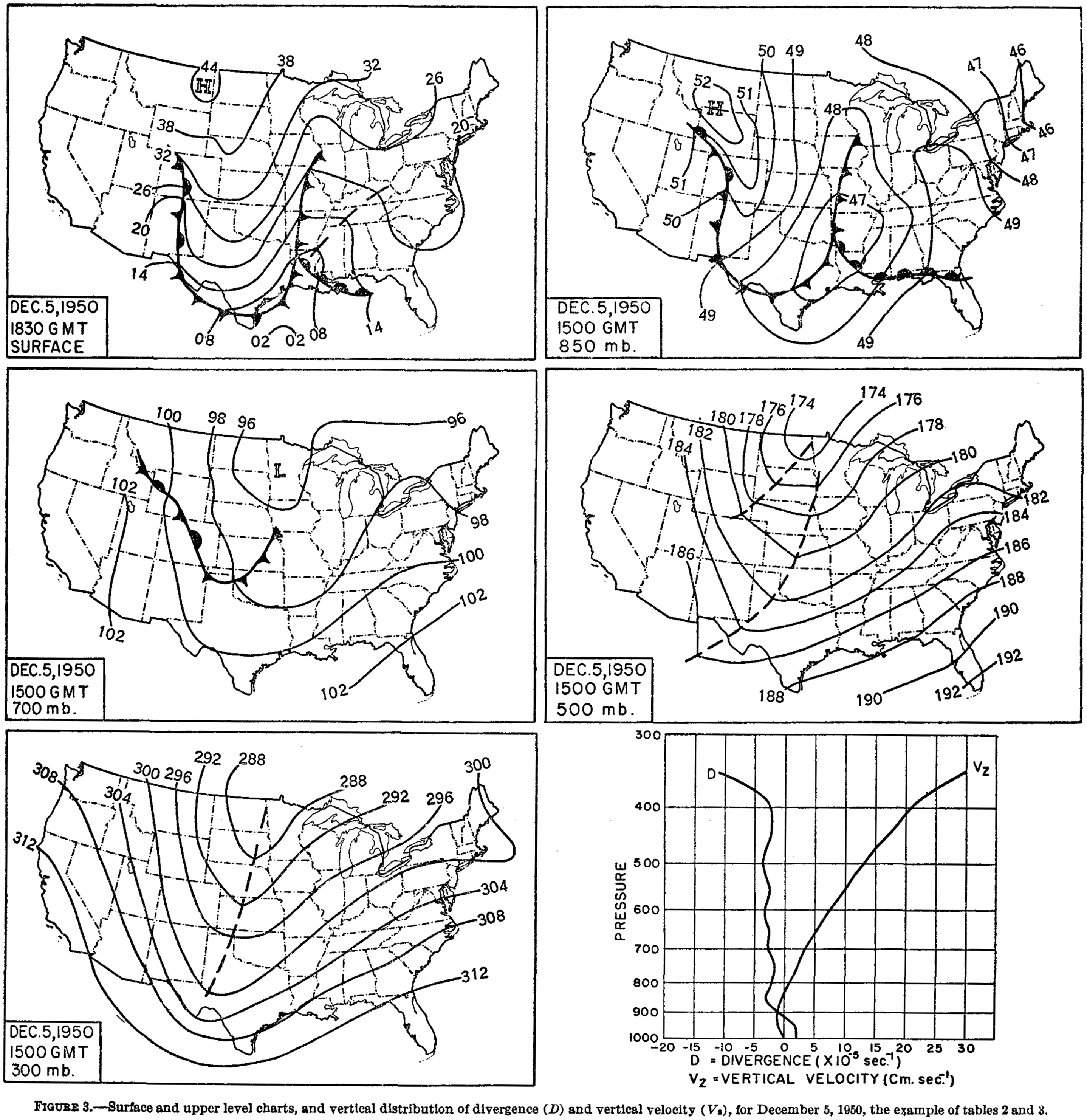

FIGURE 3.—Surface and upper level charts, and vertical distribution of divergence (D) and vertical velocity (V_z), for December 5, 1950, the example of tables 2 and 3.

dew point temperatures (>15° C.) and large vertical speeds (>5 cm. sec.$^{-1}$), errors as large as 15 percent or greater may be introduced. In such cases actual condensation temperatures and pressures were computed. Errors caused by these assumptions always cause the computed amounts to be higher than if actual condensation temperatures and pressures were used.

Using these assumptions, a procedure for computing the rate of rainfall for each layer may be programmed and the summation indicated by equation (2) performed in order to give the total precipitation during the 12-hour period. Table 3 is an example of these computations.

It will be noted, in this instance, that the precipitation was computed as originating from levels above 750 mb., with the maximum amount being contributed by the layer centered at 600 mb. The RAWIN data terminated with the 350-mb. layer, where an extremely large value of convergence (and thus vertical speed) was calculated. Such anomalies in the pattern of vertical motion were not infrequently observed at levels above 500 mb., but it is impossible to conclude from these data whether or not the effects are real and perhaps associated with upper level jet phenomena, or are fictitious and caused by unavoidable instrumental errors in the measurement of strong winds at high elevations. In any case, because of the small amounts of moisture at high levels the contribution to the total precipitation was usually small and, as was pointed out previously, could usually be neglected.

As a matter of interest, the synoptic meteorological conditions for this example are shown in figure 3. It will be seen that the computed vertical distribution of convergence-divergence and vertical motion associated with this situation seems quite reasonable when compared with the synoptic models usually thought to apply in the atmosphere.

SUMMARY OF RESULTS

Table 4 presents the results of carrying out these computations for the winter months of December 1949, and January, February, and December 1950. These data are summarized in table 5 for all cases where RAWIN-RAOB data were available above the 700-mb. level. Considering the 179 nonoverlapping 12-hour periods for which computations could be made, the results showed a skill of 27 percent above chance for the four precipitation classes shown in table 5. A Chi-square test of these data showed that there was less than one chance out of a hundred that this distribution could have been obtained by randomly selected data.[3] Accordingly, while the procedure outlined in the preceding discussion cannot, perhaps, be considered sufficiently precise for practical use in its present form, the results indicate (*a*) that the physical concepts used are statistically verified with a high degree of confidence when applied to the actual atmosphere, and (*b*) that upper air observations are sufficiently accurate to provide a measure of vertical velocity and moisture which can be used in computing precipitation intensity.

[3] It should be noted that the validity of the Chi-square test when applied to these data is somewhat doubtful because of the probable serial correlation between successive observations, as well as for other reasons. However, the auto-correlation coefficient for a one-period (12 hour) lag in the observed precipitation is only 0.4 so that this effect is not likely to influence the conclusion to any great degree. In any event, this serial correlation does not affect the magnitude of the relationship (correlation) between computed and observed precipitation shown in table 5.

TABLE 4.—*Computed (C) and observed (O) precipitation amounts in the Columbia-Nashville-Little Rock area. M=RAWIN and/or RAOB data missing at 700-mb. level and above. Amounts in parentheses indicate computations made when RAWIN/RAOB data terminated between 700 mb. and 500 mb. T indicates trace (less than 0.005 inch)*

Date	December 1949				January 1950				February 1950				December 1950			
	03-15 GMT		15-03 GMT		03-15 GMT		15-03 GMT		03-15 GMT		15-03 GMT		03-15 GMT		15-03 GMT	
	C	O	C	O	C	O	C	O	C	O	C	O	C	O	C	O
1	0	0	0.27	0	T	0.30	0.06	0.09	(0)	0.42	M	0.07	0	0.01	M	T
2	(0)	0	0	0	M	.63	M	.69	(0.07)	.05	0.01	T	M	0	M	0.41
3	0	0	(0)	0.03	M	.14	M	.65	0	0	0	0	M	.33	0.06	T
4	0.22	0.20	(0)	0	M	1.24	M	.44	(0)	0	(.02)	0	M	0	M	.02
5	(0)	0	M	0	M	.33	M	.03	0	0	0	0	0.08	.03	.24	.15
6	0	0	(.15)	.13	M	.21	(0)	.06	(.14)	.01	.01	.01	.14	.33	M	.06
7	M	0	(0)	0	0	.01	0	.01	(0)	0	0	0	M	0	(T)	0
8	.01	0	T	0	(0)	0	0	0	0	.01	.31	.04	(0)	0	T	0
9	T	0	0	.01	M	T	M	.02	M	.08	0	0	(0)	0	M	0
10	0	.14	0	.41	M	.89	M	.01	0	0	0	0	(0)	0	(0)	T
11	M	.94	M	.64	0	.01	0	.06	M	0	T	.04	0	T	.14	.03
12	M	.13	(0)	.19	0	.36	(0)	.29	M	1.45	(0)	1.21	(0)	0	(0)	0
13	.58	.02	.06	0	M	.57	M	.52	.09	.94	.16	.28	.04	0	(.06)	0
14	(0)	0	.06	0	M	T	0	T	M	.24	(0)	T	M	0	0	0
15	T	0	0	0	(0)	.26	M	.39	(0)	T	T	0	(0)	0	(0)	0
16	0	T	0	T	M	.02	0	T	0	0	0	0	M	0	0	0
17	0	.03	0	.56	0	0	M	.04	M	0	0	0	0	0	0	0
18	M	.24	0	0	M	.05	.08	.03	0	T	M	.01	(0)	0	0	0
19	0	0	T	0	M	0	0	0	0	0	0	0	0	0	0	0
20	0	0	(.02)	.01	0	0	0	0	0	0	0	0	0	0	0	0
21	(.05)	.04	M	.33	0	0	0	0	0	.07	.37	.39	(0)	0	.01	0
22	M	.27	M	0	0	.01	0	0	M	.27	(.05)	T	.08	0	0	0
23	(0)	0	0	0	0.06	T	.04	T	0	0	0	0	(0)	T	T	T
24	T	0	0	0	.05	0	M	T	M	.01	0	0	0	0	(0)	0
25	(0)	0	(0)	.02	M	0	M	.02	0	0	0	0	M	0	(0)	0
26	(.43)	.63	.30	.01	M	1.03	.25	.32	(0)	0	(0)	0	M	T	(0)	0
27	(.06)	.02	M	0	M	0	(0)	.01	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	M	T	0	.38	M	.16	0	0	0	0
29	0	0	0	0	(.02)	.04	(.30)	T	------	------	------	------	M	0	T	0
30	0	0	0	0	M	.05	(0)	.47	------	------	------	------	0	0	0	0
31	0	.02	0	.03	M	.16	(0)	.59	------	------	------	------	0	0	0	0

TABLE 5.—*Summary of computed vs. observed precipitation amounts from table 4. Data include all cases except those with missing (M) RAWIN observations.*

		Computed				
		0	0.01–0.25	0.26–0.50	>0.50	Total
Observed	0	110	14	2	0	126
	0.01–0.25	19	14	2	1	36
	0.26–0.50	8	3	1	0	12
	>0.50	3	1	1	0	5
	Total	140	32	6	1	179

Percent correct: $\frac{125}{179}=0.70$

Skill score: $\frac{125-105}{179-105}=0.27$

Chi-square (1 degree freedom): 20.8*

*This value was computed for a 2-by-2 table in order to combine frequencies so that expected values would exceed 5.

An examination of table 4 will show that, on many of the days with moderate or heavy precipitation, no computations could be made, due primarily to missing RAWIN data. By far the greatest number of these missing wind observations were caused by "limiting angles", i. e., by the balloon dropping below the critical 15° elevation angle where present RAWIN equipment (SCR–658) cannot be relied upon to give accurate measurements. From a study of individual ascents, it appeared that these limiting angles were due to (*a*) strong winds aloft predominantly from a single general direction and/or (*b*) slow ascensional rates, the latter probably being caused partly by lack of sufficient initial free lift being imposed on the balloon by the observer, and partly by the added drag of the precipitation during flight. This study therefore emphasizes both the importance of care in making observations and the need for improved instrumental equipment to alleviate the technical difficulty in obtaining winds aloft measurements.

It will also be observed that there appears to be a tendency for the computed amounts to be somewhat smaller than those observed. This bias is probably due in part to the neglect of such factors as radiational cooling of clouds, or to the fact that only large-scale precipitation mechanisms are considered. This latter effect is of particular interest, for it can be shown rather simply that, if upward small-scale motions are accompanied by compensatory downward motions within an area of large-scale circulation, the average precipitation over the area covered by the large-scale circulation will always be greater than that accounted for by considering the large-scale effects alone. This means that the computed rainfall amounts associated with thunderstorms, for example, will in general be too small, even when averaged over a large area, if the computations are based on the large-scale features. In an attempt to take this effect into account, the writers are currently engaged in an investigation of data from the Thunderstorm Project [13].

CONCLUSION

In the preceding discussion there has been described a method of attack on the problem of computing, from the currently observed fields of moisture and wind, the expected intensity of precipitation during the 12-hour period between RAWIN-RAOB observations. A physical, and therefore quite general, approach has been developed. A test of the procedure applied to the actual atmosphere has provided statistical evidence of the usefulness of the method, the validity of the assumptions made, and the accuracy and representativeness of the observational data when used for this purpose.

It should be pointed out, however, that this discussion represents only an exploratory study of the larger problem of computing precipitation from currently made prognostic charts. Such prognoses at present contain, in addition to the frontal analyses, only the height of the pressure surface. This means that the wind field must be obtained from an approximation, and that the moisture field cannot be obtained at all. It is desired to point out that a knowledge of the latter is of considerable importance in precipitation forecasting and a study of its prediction and eventual inclusion on the regularly issued prognostic charts should accordingly be undertaken. The effect of using a wind approximation obtained from the contour field is not yet known, but work on this problem is currently being undertaken by the writers. These studies include the use of the "gradient" wind instead of the "true" wind in the preceding method of computing divergence, a procedure for obtaining the divergence from the geostrophic vorticity using the vorticity tendency equation, and certain other devices which have been suggested. Preliminary results indicate that the first of these procedures is of little practical usefulness, but the use of the geostrophic vorticity in the vorticity tendency equation seems to provide estimates of the precipitation intensity which are, at least in a preliminary way, somewhat superior to those given in this paper. This may, as was pointed out earlier, be due to the ability of the geostrophic vorticity to filter out small scale circulations which are present in the "true" winds, and thus to reduce the random errors introduced by the small scale "noise."

Although the integration was, in this case, carried out for 10 layers using increments of 50 mb. in thickness, it is probable that useful results could be obtained from fewer layers. In an incidental investigation, using a group of the same data included in table 4 but making calculations with only the standard levels for which upper air charts are now prepared, no significant change in the accuracy of the precipitation computation was observed. This means, that, from an operational standpoint, accurate prognostic information limited to, say, 4 or 5 levels would probably be sufficient to provide useful results.

ACKNOWLEDGMENT

The writers would like to express their appreciation to Mr. Roger A. Allen and the members of the Short Range Forecast Development Section for valuable suggestions contributed in discussions held jointly and severally during the progress of the study. Credit is also due to Miss Olga Knapp for performing the greater part of the calculations required.

REFERENCES

1. J. G. Charney, "Dynamic Forecasting by Numerical Process", *Compendium of Meteorology,* American Meteorological Society, 1951, pp. 470–482.
2. L. Carstensen and A. Hardy, Use of Prognostic Charts in Objective Precipitation Forecasts for New York City, U. S. Weather Bureau, Washington, D. C., April 1951 (unpublished).
3. C. L. Bristor, Relating Prognostic Charts to Winter Precipitation at Sioux City, Iowa, U. S. Weather Bureau, Washington, D. C., January 1952 (unpublished).
4. F. Möller, "Long Wave Radiation", *Compendium of Meteorology,* American Meteorological Society, 1951, pp. 34–49.
5. T. E. Hoffer, "A Study of Long Wave Radiation Balance in Certain Typical Air Masses," Atmospheric Radiation Project, *Quarterly Progress Report No. 5,* University of Utah, Meteorology Dept. September 1951.
6. F. Möller, "Thermodynamics of Clouds," *Compendium of Meteorology,* American Meteorological Society, 1951, pp. 199–206.
7. J. R. Fulks, "Rate of Precipitation from Adiabatically Ascending Air," *Monthly Weather Review,* vol. 63, No. 10, October 1935, pp. 291–294.
8. H. R. Byers and H. R. Rodebush, "Causes of Thunderstorms of the Florida Peninsula," *Journal of Meteorology,* vol. 5, No. 6, December 1948, pp. 275–280.
9. J. E. Miller, "*Application of Vertical Velocities to Objective Weather Forecasting,*" New York University, Dept. of Meteorology, 1946, 85 pp.
10. P. K. Das, "On the Use of Convergence Charts over the Indian Region," *Indian Journal of Meteorology and Geophysics,* vol. 2, No. 3, July 1951, pp. 172–179.
11. J. K. Bannon, "The Estimation of Large-Scale Vertical Currents from the Rate of Rainfall," *Quarterly Journal of the Royal Meteorological Society,* vol. 74, No. 319, January 1948, pp. 57–66.
12. J. C. Bellamy, "Objective Calculations of Divergence, Vertical Velocity, and Vorticity," *Bulletin of the American Meteorological Society,* vol. 30, No. 2, February 1949, pp. 45–49.
13. H. R. Byers and R. R. Braham, Jr., *The Thunderstorm,* U. S. Weather Bureau, Washington, D. C., 1949, 287 pp.